COHERENCE AND SYNCHRONIZATION IN DIODE–LASER ARRAYS WITH DELAYED GLOBAL COUPLING

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Abstract

The dynamics of a semiconductor–laser array whose individual elements are coupled in a global way through an external mirror is numerically analysed. A coherent in–phase solution is seen to be preferred by the system at intermediate values of the feedback coupling strength. At low values of this parameter, a strong amplification of the spontaneous emission noise is observed. A tendency towards chaos synchronization is also observed at large values of the feedback strength.

RUNNING TITLE: Coherence and synchronization in laser arrays

Semiconductor lasers are used in a wide range of applications including, among other examples, optical communication systems, compact disk and CD-ROM units, bar–code readers, and laser printers. Some of their advantageous features are their small size, high efficiency, low price, simple pumping requirements, and the possibility to modulate their injection current at high frequencies. In general, semiconductor lasers can emit either in a continuous or in a pulsating way, but in any case the maximum emitted power is usually lower than a few tens of mW. However, for many applications it is desirable to have high–power coherent emission available from small sources. So far, edge–emitting semiconductor lasers have been designed to produce relatively high power (up to a few W) by arranging them in one–dimensional arrays [Welch *et al.*, 1988]. More recently, two–dimensional arrays of vertical–cavity surface–emitting lasers (VCSELs) have also been developed [Sanders *et al.*, 1994]. Increase in the output power is obtained through either coherent or incoherent superposition of the laser beams emitted by the individual elements in the array. In the coherent case, the phase difference between neighboring beams has a constant value (the array is said to be *phase locked*). This situation is usually preferred, since in this case the maximum output power scales with the square of the number of elements in the array. On the other hand, an incoherent (non phase–locked) superposition leads to a total output power whose maximum value scales linearly with the number of elements.

In order to have phase locking, some kind of coupling must exist between the dynamics of the individual lasers of the array. In standard diode–laser arrays, coupling between the elements occurs *locally* through the smooth overlap of the evanescent field of each individual laser with those of its nearest neighbours. In this case, the laser array can reach a phase–locked state if the separation between neighboring elements is small enough (typically lower than 10 μm). Phase locking usually arises in one of two ways: either all lasers emit with the same phase (in–phase solution) or the phase difference between neighboring lasers is fixed to a constant nonzero value (out–of–phase solution). The in–phase solution produces constructive interference in the optical axis, whereas for the out–of–phase state, interference is destructive. Local coupling usually leads to out–of–phase emission [Winful & Wang, 1988], which is more favorable energetically; in fact the stability range of the in–phase solution is seen to be very small [Li & Erneux, 1993].

In this work, we propose a different way to couple the elements of the array, with the aim of favoring the in–phase coherent state. Instead of using a local coupling like the one provided by the evanescent field, we now consider a global, all–to–all coupling through an

external mirror at one side of the laser [Leger *et al.*, 1988]. Global coupling is known to enlarge the stability range of the in–phase solution [Li & Erneux, 1993]. With such a configuration, a coherent superposition of the individual laser beams takes place at the mirror, and part of the resulting electromagnetic field is fed back into the array, in such a way that each individual laser is influenced by all other elements of the array in an all–to–all coupling. A characteristic feature of this global coupling is that it involves a delay, due to the time τ that the light takes to propagate towards the mirror and back into the array. Other feedback schemes have been recently proposed to stabilize phase locking [Dasgupta & Andersen, 1994] and to control the chaotic output [Auerbach & Yorke, 1996] of semiconductor laser arrays. In those cases, however, the feedback loops are of optoelectronic nature, whereas our current proposal involves all–optical feedback coupling. *Local* feedback coupling has also been recently proposed to control the transverse dynamics of broad–area lasers [Martin–Regalado *et al.*, 1996].

In a first approximation, we ignore the evanescent local coupling by considering a large enough separation between neighboring lasers of the array. The array output is observed at the focal plane of a lens (at the opposite side of the external mirror), which is called the *far–field* plane and corresponds to the Fourier transform of the system.

In order to analyse the behavior of the proposed scheme, we have performed numerical simulations of a system of coupled rate–equations based on the Lang–Kobayashi model [Lang & Kobayashi, 1980]:

$$\frac{dE_i}{dt} = \frac{1+i\alpha}{2}\left(G_i(E_i,N_i)-\gamma\right)E_i(t) + \kappa e^{-i\omega\tau}\sum_{j=1}^{n} E_j(t-\tau) + \sqrt{2\beta N_i}\xi_i(t) \tag{1}$$

$$\frac{dN_i}{dt} = \gamma_e\left(CN_{th}-N_i\right) - G_i(E_i,N_i)\left|E_i(t)\right|^2 \tag{2}$$

where n is the number of lasers in the array, $E_i(t)$ is the complex envelope of the electric field emitted by laser i, and N_i is the corresponding carrier number. The material gain is given by

$$G_i(E,N) = \frac{g(N_i(t)-N_0)}{1+s|E_i(t)|^2}$$

and the threshold value of the carrier number is $N_{th} = \gamma/g + N_0$. Feedback is described by two parameters, the feedback strength κ and the external round–trip time τ. For simplicity, we have assumed that both κ and τ are the same for all the lasers of the array. The feedback term also depends on the frequency $\omega = 2\pi c/\lambda$, where λ is the wavelength of the emitted light and $c = 3\times10^8$ m/s is the speed of light in vacuum. The random spontaneous emission process is modeled by a complex Gaussian white noise term $\xi(t)$ of zero mean and correlation

$\langle \xi(t)\xi^*(t')\rangle = 2\delta(t-t')$. The definitions and values of the rest of parameters are given in Table 1. Equations (1)–(2) are numerically integrated by means of a second–order Runge–Kutta algorithm for the deterministic terms and a stochastic Euler method for the noise terms.

Parameter	Description	Value
C	relative bias current	2.0
γ_e	inverse carrier lifetime	5×10^8 s^{-1}
γ	inverse photon lifetime	5×10^{11} s^{-1}
α	linewidth enhancement parameter	5
λ	laser wavelength	1.5 μm
g	differential gain coefficient	1.5×10^4 s^{-1}
N_0	transparency inversion	1.5×10^8
s	saturation coefficient	10^{-6}
β	noise intensity	10^3 s^{-1}

Table 1: Parameters of the diode–laser array model.

In order to characterize the behavior of the system, we define an incoherent intensity

$$S = \sum_{i=1}^{n} |E_i(t)|^2 \tag{3}$$

and a coherent intensity

$$I = \left| \sum_{i=1}^{n} E_i(t) \right|^2 \tag{4}$$

The physical interpretation of these magnitudes is the following. S is the sum of the individual laser intensities; it can be measured by placing a broad–area detector next to the output face of the array, and it is defined in such a way that S/n gives the output level of emission from one individual laser. The intensity I, on the other hand, corresponds to a coherent superposition of the individual electric fields; it can be measured by placing a detector at the far–field plane of the lens.

Typical phase–locked states of the system are shown in Fig. 1, for a three–laser array and a delay time $\tau = 500$ ps. At low values of the feedback strength κ an out–of–phase solution arises (Fig. 1a), characterized by a value of I close to 0 (destructive interference); in fact the intensity profile in the far field has in this case a double–lobed structure with

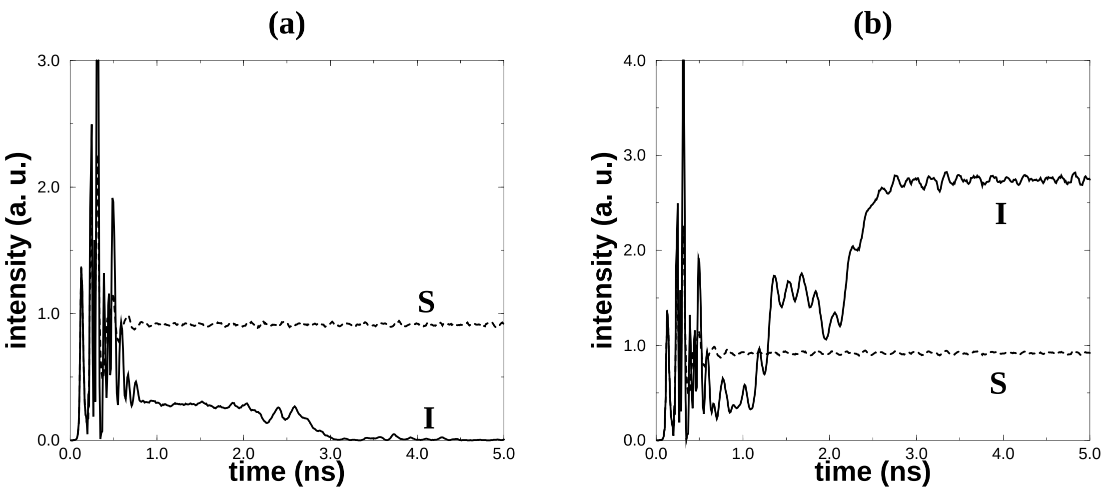

Figure 1: Phase locked behavior of the diode–laser array for $\tau = 500$ ps. (a) Out–of–phase solution for $\kappa = 10^{-4}$ ps^{-1}; (b) in–phase solution for $\kappa = 9 \times 10^{-4}$ ps^{-1}.

a minimum at the optical axis. We have also observed that the phase difference between neighboring lasers is $2\pi/3$.

Contrarily to what happens in locally coupled arrays, an in–phase solution can now be easily obtained by increasing the feedback strength. Figure 1b shows such an in–phase behavior, for which $I \approx nS$ (which means that the intensity at the far–field plane is n^2 times the output from a single laser); the intensity profile in the far field is now single lobed. The phase difference between neighboring lasers is 0. We have checked that in most of the cases we tested, different values of the κ parameter for different lasers of the array still give in–phase solution, although the output power can be lower than in the case of equal κ. A detailed study of the synchronization dependence on feedback parameters is left for a near future. We should note that the range of stability of this in–phase solution is very broad; a transition from the out–of–phase to the in–phase state can be easily induced simply by increasing κ. Preliminary results show that this transition is hysteretical, which indicates the possibility of coexistence between the in–phase and out–of–phase coherent states. The stability region of the in–phase state depends on the external round–trip time τ. In particular, simulations show that for values of τ not too large, the width of this region increases when τ decreases. Additionally, and according to what is known in the case of a single semiconductor laser with optical feedback, the phase accumulated by the field in the external cavity, $\phi_{ext} = \omega\tau$, should also play an important role in the behavior of the system. For the parameters that we have chosen and for $\tau = 500$ ps, this accumulated phase is ϕ_{ext}=0 (mod$_{2\pi}$). We have

checked that up to values $\phi_{ext} \sim \pm 0.5$ (mod$_{2\pi}$), in–phase coherence is still well reached. A more detailed study of this effect will be carried out in the future.

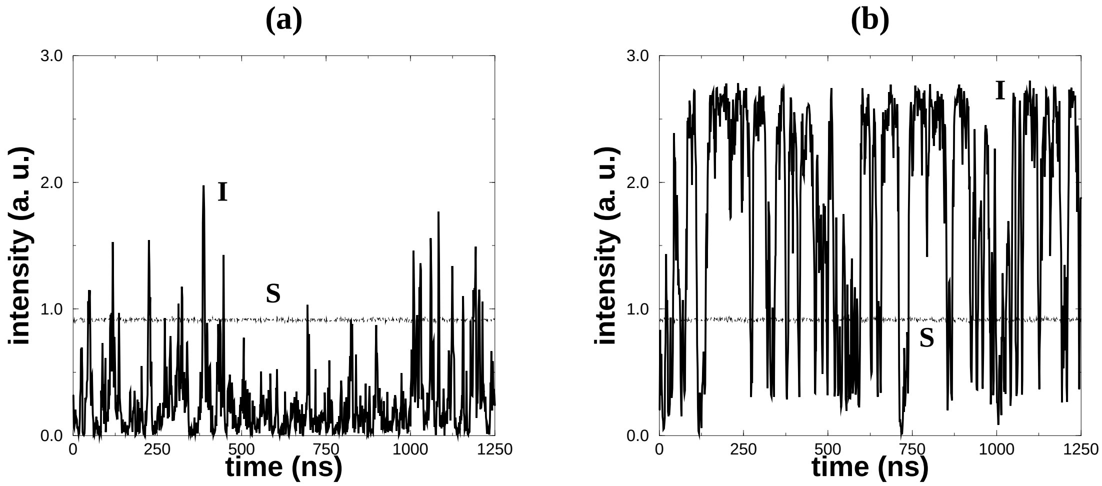

Figure 2: Incoherent bursts of the dynamics for $\kappa = 5 \times 10^{-5}$ ps^{-1}. (a) Departure from the out–of–phase state for $\tau = 250$ ps; (b) fluctuating jumps between in–phase and out–of–phase behavior for $\tau = 125$ ps. This phenomenon disappears if spontaneous–emission noise is not considered in the model.

For small values of the feedback strength κ, the delay time τ starts to have an important role in the dynamics of the system. For τ large enough ($\tau \sim 1$ ns) the system develops an out–of–phase coherent state such as the one shown in Fig. 1a. However, when τ decreases bursts of incoherent behavior start to appear, leading to a highly fluctuating value of the coherent intensity I. This situation is shown in Fig. 2 for $\kappa = 5 \times 10^{-5}$ ps^{-1} and two small values of the delay time τ. For $\tau = 250$ ps (Fig. 2a), high–amplitude bursts of incoherent behavior appear from the out–of–phase state. This highly fluctuating dynamics intensifies for smaller τ: for $\tau = 125\, ps$ (Fig. 2b), the system jumps wildly between the in–phase and out–of–phase states. A naïve interpretation of this behavior can be given by reminding that a decreasing delay time tends to increase the stability range of the in–phase solution. This tendency contrasts with the destabilizing role of a decreasing feedback strength. A competition between these two effects could be the reason for the highly fluctuating behavior of the system. It should also be noted that this complex phenomenon only occurs if spontaneous emission noise (as described by the stochastic term $\xi_i(t)$ in Eq. (1)) is considered. Hence it can be said that one is observing an intense noise amplification for low values of κ and τ.

Finally, for large values of the feedback coupling strength κ, the array undergoes chaotic

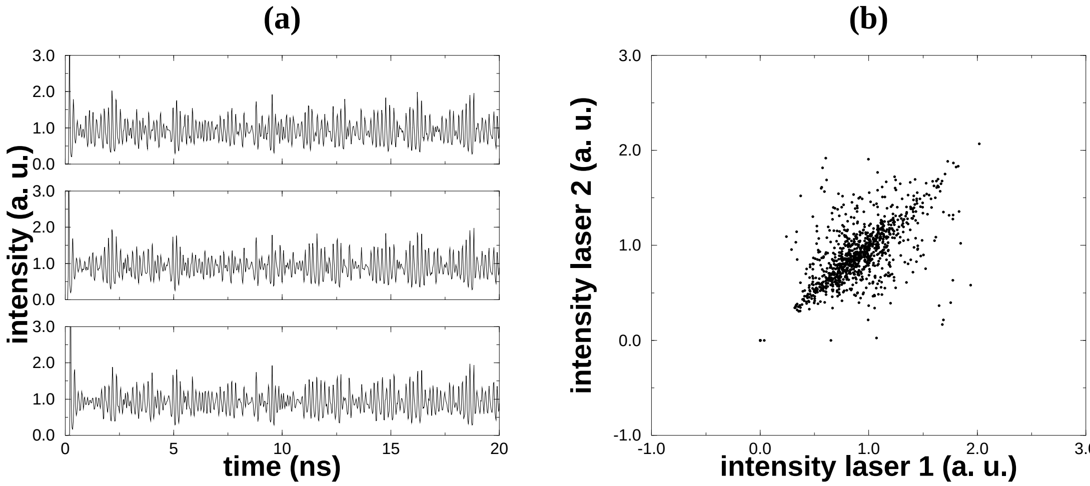

Figure 3: Chaotic synchronization in a three–laser array for large feedback strength $\kappa = 0.002$ ps^{-1} and $\tau = 500$ ps. (a) Chaotic oscillations of the individual lasers; (b) synchronization plot of laser 1 vs. laser 2.

oscillations, as shown in Fig. 3 for a three–laser array with $\kappa = 0.002$ ps^{-1} and $\tau = 500$ ps. This situation is to be expected, since single diode lasers exhibit a rich chaotic dynamics in the presence of optical feedback [Sacher *et al.*, 1992]. Indeed, each individual laser of the array behaves chaotically, as shown in Fig. 3a, whereas some tendency towards synchronization can be observed by plotting the output intensity of one of the lasers versus another (Fig. 3b). Chaos synchronization in laser arrays was predicted numerically by Winful & Rahman [1990] and observed experimentally by Roy & Thornburg [1994].

In conclusion, we have numerically analysed the dynamical behavior of a semiconductor laser array in which the individual elements are coupled in a *global* way through an optical feedback. The system is seen to select an in–phase coherent solution for intermediate values of the feedback coupling strength κ. For low values of κ, noise amplification is observed in the form of random bursts of incoherent behavior. Finally, for large values of κ the dynamics of the individual lasers is chaotic, and some tendency towards chaos synchronization is observed.

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REFERENCES

Auerbach, D. & Yorke, J.A. [1996] "Controlling chaotic fluctuations in semiconductor laser arrays," *J. Opt. Soc. Am. B* **13**(10), 2178–2187.

Dasgupta, S. & Andersen D.R. [1994] "Feedback stabilization of semiconductor laser arrays," *J. Opt. Soc. Am. B* **11**(2), 290–296.

Lang, R. & Kobayashi, K. [1980] "External optical feedback effects on semiconductor injection laser properties," *IEEE J. Quantum Electron.* **QE–16**(3), 347–355.

Leger, J.R., Scott, M.L. & Veldkamp, W.B. [1988] "Coherent addition of AlGaAs lasers using microlenses and diffractive coupling," *Appl. Phys. Lett.* **52**(21), 1771–1773.

Li, R. & Erneux, T. [1993] "Stability conditions for coupled lasers: series coupling versus parallel coupling," *Optics Commun.* **99**(3–4), 196–200.

Martin–Regalado, J., van Tartwijk, G.H.M., Balle, S. & San Miguel, M. [1996] "Mode control and pattern stabilization in broad–area lasers by optical feedback," *Phys. Rev. A* **54**(6), 5386–5393.

Roy, R. & Thornburg, K.S. [1994] "Experimental synchonization of chaotic lasers," *Phys. Rev. Lett.* **72**(13) 2009–2012.

Sacher, J., Baums, D., Panknin, P., Elsässer, W. & Göbel, E.O. [1992] "Intensity instabilities of semiconductor lasers under current modulation, external light injection, and delayed feedback," *Phys. Rev. A* **45**(3), 1893–1905.

Sanders, S., Waarts, R., Nam, D., Welch, D., Scifres, D., Ehlert, J.C., Cassarly, W.J., Finlan J.M. & Flood K.M. [1994] "High power coherent two–dimensional semiconductor laser array," *Appl. Phys. Lett.* **64**(12), 1478–1480.

Welch, D.F., Chan, B., Streifer, W. & Scifres, D.R. [1988] "High–power, 8W cw, single–quantum–well laser diode array," *Electron. Lett.* **24**(2), 113–115.

Winful, H.G. & Rahman L. [1990] "Synchronized chaos and spatiotemporal chaos in arrays of coupled lasers," *Phys. Rev. Lett.* **65**(13), 1575–1578.

Winful, H.G. & Wang S.S. [1988] "Stability of phase locking in coupled seminconductor laser arrays," *Appl. Phys. Lett.* **53**(20), 1894–1896.